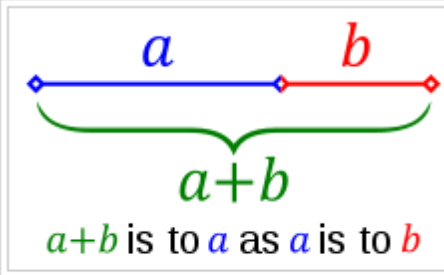
Golden Ratio

Introduction

In mathematics and the arts, two quantities are in the **golden ratio** if the ratio of the sum of the quantities to the larger quantity is equal to the ratio of the larger quantity to the smaller one.

The golden ratio is an **irrational** mathematical constant, approximately 1.61803398874989. Other names frequently used for the golden ratio are the golden section and golden mean. Other terms encountered include extreme and mean ratio, medial section, divine proportion, divine section, golden proportion, golden cut, golden number, and mean of Phidias. In this article, the golden ratio is denoted by the Greek lowercase letter phi (φ)



The golden section is a line segment divided according to the golden ratio: The total length $a + b$ is to the length of the longer segment a as the length of a is to the length of the shorter segment b.

At least since the Renaissance[1], many artists and architects have proportioned their works to approximate the golden ratio—especially in the form of the **golden rectangle** (see Golden Rectangle). Mathematicians have studied the golden ratio because of its unique and interesting properties. The golden ratio is also used in the analysis of financial markets.

History

The golden ratio has fascinated Western intellectuals of diverse interests for at least 2,400 years. According to Mario Livio:

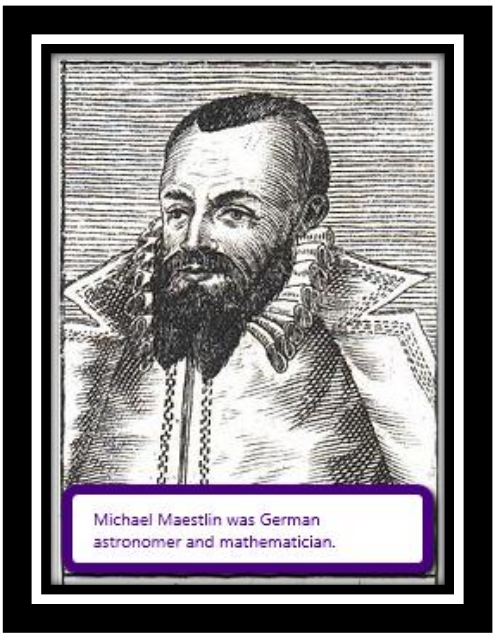
> Some of the greatest mathematical minds of all ages, from Pythagoras and Euclid in ancient Greece, through the medieval Italian mathematician Leonardo of Pisa and the Renaissance astronomer Johannes Kepler, to present-day scientific figures such as Oxford physicist Roger Penrose, have spent endless hours over this simple ratio and its properties. But the fascination with the Golden Ratio is not confined just to mathematicians. Biologists, artists, musicians, historians, architects, psychologists, and even mystics have pondered and debated the basis of its ubiquity[2] and appeal. In fact, it is probably fair to say that the Golden Ratio has inspired thinkers of all disciplines like no other number in the history of mathematics.

Ancient Greek mathematicians first studied what we now call the golden ratio because of its frequent appearance in geometry. The division of a line into "extreme and mean ratio" is important in the geometry of

[1] 14th and 17th Century Europe marking the transition from the medieval to the modern world
[2] the state of being everywhere

regular pentagrams and pentagons. The Greeks usually attributed discovery of this concept to Pythagoras or his followers.

Euclid's *Elements* provides the first known written definition of what is now called the golden ratio. Throughout the Elements, several propositions[3] and their proofs employ the golden ratio. Some of these propositions show that the golden ratio is an irrational number.



The name "extreme and mean ratio" was the principal term used from the 3rd century BC until about the 18th century.

Michael Maestlin, first to publish a decimal approximation of the golden ratio, in 1597.

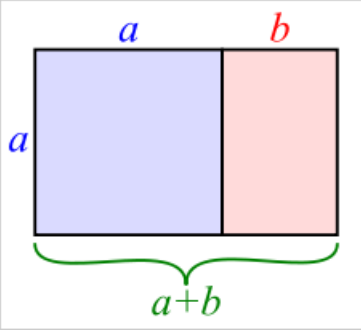
Since the twentieth century, the golden ratio has been represented by the Greek letter ***Φ*** or ***φ*** (phi, after Phidias, a sculptor who is said to have employed it) or less commonly by ***τ*** (tau, the first letter of the ancient Greek root τομή—meaning *cut*).

Golden Rectangle

A **golden rectangle** is one whose side lengths are in the golden ratio:

$1 : \frac{1+\sqrt{5}}{2}$ or approximately 1:1.618

A distinctive feature of this shape is that when a square section is removed, the remainder is another golden rectangle; that is, with the same proportions as the first. Square removal can be repeated infinitely, in which case corresponding corners of the squares form an infinite sequence of points on the golden spiral.



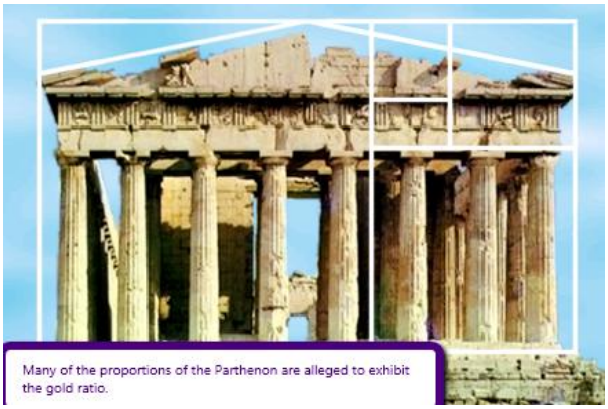
A golden rectangle with longer side *a* and shorter side *b*, when placed adjacent to a square with sides of length *a*, will produce a similar golden rectangle with longer side *a* + *b* and shorter side *a*. This illustrates the relationship $\frac{a+b}{a} = \frac{a}{b} \equiv \varphi\,.$

[3] rules in modern terminology

Applications & Observations

Architecture

The Parthenon's facade as well as elements of its facade and elsewhere are said by some to be circumscribed[4] by golden rectangles. Other scholars deny that the Greeks had any aesthetic[5] association with golden ratio.


Many of the proportions of the Parthenon are alleged to exhibit the gold ratio.

Paintings

A statistical study on 565 works of art of different great painters, performed in 1999, found that these artists had not used the golden ratio in the size of their canvases. The study concluded that the average ratio of the two sides of the paintings studied is 1.34, with averages for individual artists ranging from 1.04 (Goya) to 1.46 (Bellini). On the other hand, Pablo Tosto listed over 350 works by well-known artists, including more than 100 which have canvasses with golden rectangle and root-5 proportions, and others with proportions like root-2, 3, 4, and 6.

Industrial Design

Some sources claim that the golden ratio is commonly used in everyday design, for example in the shapes of postcards, playing cards, posters, wide-screen televisions, photographs, and light switch plates.

[4] encircled
[5] having a sense of beauty